TAKING ACCOUNT OF SLIP AND CONVECTION IN THE GAS BETWEEN TWO PARALLEL PLATES

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The flow mode of a rarefied gas between two parallel plates with a sinusoidal temperature distribution on one of the plates is investigated. Slip and free convection are taken into account.

Let us consider the flow of a weakly rarefied gas between two parallel plates with a sinusoidal temperature distribution on one of the plates and taking account of slip and free convection.

Let us select X and Y axes, respectively, along and normal to the lower plate surface. Let us consider the amplitude of the temperature change α and the ratio between the mean free path l and the wavelength of the temperature change L to be small. Let us seek the temperature distribution T, the density ρ, and the pressure p of the gas in the form

$$T = T_0(1+\tau), \quad \rho = \rho_0(1+\sigma), \quad p = p_0(1+\xi),$$

where T_0 and ρ_0 correspond to $\alpha = 0$ and $p_0 = p_1 - \rho_0 gY$.

The velocities u and v and the quantities τ, σ, and ξ are small. The linearized system of equations describing the free convection is the following [1]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\mu \Delta u = \frac{L}{2\pi} p_0 \frac{\partial \xi}{\partial x}, \tag{2}$$

$$\mu \Delta v = -\frac{\rho_0 g L^2}{(2\pi)^2}\xi + \frac{L}{2\pi} p_0 \frac{\partial \xi}{\partial y} - \frac{\beta \rho_0 g T_0 L^2}{(2\pi)^2}\tau, \tag{3}$$

$$\Delta \tau = 0, \tag{4}$$

$$\sigma = -\beta T_0 \tau. \tag{5}$$

Here

$$x = \frac{2\pi X}{L}, \quad y = \frac{2\pi Y}{L}.$$

In the case of a sinusoidal temperature change on the lower plate ($\tau_W = \alpha \sin x$) the boundary conditions are (we consider $\exp\{-(2\pi d/L)\}$ to be negligibly small and the perturbation does not reach the upper plate):

for $y = 0$ [2]

$$u = b_1 \frac{\partial u}{\partial y} + b_2 \frac{\partial \tau}{\partial x}, \tag{6}$$

$$v = 0, \tag{7}$$

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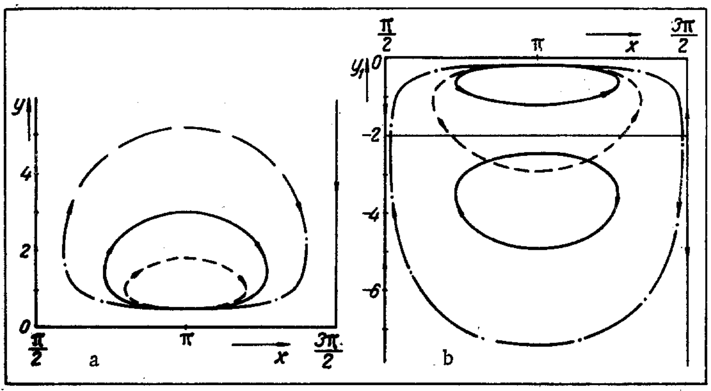

Fig. 1. Solid streamlines for a correspond to (17), and for b to (17'), while the dashes correspond to E = 0 for a and b; the dash-dot lines correspond to k = 0 for a and k_1 = 0 for b.

$$\tau - \tau_w = c\frac{\partial \tau}{\partial y}, \tag{8}$$

where

$$b_1 = \frac{2\pi(2-f)}{f}\frac{l}{L}, \quad b_2 = \frac{3}{2}(2\pi R T_0)^{1/2}\frac{l}{L},$$

$$c = \frac{15\pi(2-f)T_0}{4f}\frac{l}{L},$$

for y = $(2\pi d/L)$ = h (or for $y \to \infty$)

$$u = 0, \quad v = 0, \quad \tau = 0, \quad \xi = 0. \tag{9}$$

Solving (4) with (9) taken into account, we find

$$\tau = a \exp\{-y\}\sin x, \tag{10}$$

where the constant a is determined from condition (8)

$$a = \alpha(1 - c). \tag{11}$$

Eliminating u and v from (1)-(3), we obtain the equation

$$\Delta\xi - \frac{\rho_0 g L}{\pi p_0}\frac{\partial \xi}{\partial y} = \frac{\beta\rho_0 g T_0 L}{2\pi p_0}\frac{\partial \tau}{\partial y}. \tag{12}$$

Using (10), let us solve (1), (2), and (12) with conditions (7) and (9):

$$\xi = \left[-\frac{\beta T_0 a}{2} + \frac{2\pi\mu}{L p_0}\left(2k - D - \frac{3}{2}E\right)\right]\exp\{-y\}\sin x, \tag{13}$$

$$u = \left[k + \left(\frac{E}{2} - k\right)y - \frac{E}{4}y^2\right]\exp\{-y\}\cos x, \tag{14}$$

$$v = \left(ky + \frac{E}{4}y^2\right)\exp\{-y\}\sin x, \tag{15}$$

where

$$D = -\frac{\beta T_0 a p_1 L}{4\pi\mu}; \quad E = \frac{\rho_0 g L^2 \beta T_0 a}{8\pi^2\mu},$$

and the coefficient k is determined from condition (6):

$$k = \frac{b_1 E}{2} + b_2 a. \tag{16}$$

On the basis of (14) and (15), let us introduce the stream function

$$\psi = \left(ky + \frac{E}{4} y^2 \right) \exp\{-y\} \cos x. \quad (17)$$

Let us transfer the origin of the coordinate system for a sinusoidal change in the temperature on the upper plate by introducing $y_1 = y - h$. In this case the Eqs. (1)-(5) are retained, conditions (9) are satisfied for $y_1 = -h$ and conditions (6)-(8) for $y_1 = 0$ ($\partial/\partial y$ is replaced by $-\partial/\partial y_1$ in (6) and (8)).

We obtain the following results:

$$\tau = a \exp\{y_1\} \sin x, \quad (10')$$

$$\xi = \left[-\frac{\beta T_0 a}{2} + \frac{2\pi\mu}{L p_0} \left(2k_1 - D_1 + \frac{3}{2} E \right) \right] \exp\{y_1\} \sin x, \quad (13')$$

$$u = \left[k_1 + \left(k_1 + \frac{E}{2} \right) y_1 + \frac{E}{4} y_1^2 \right] \exp\{y_1\} \cos x, \quad (14')$$

$$v = \left(k_1 y_1 + \frac{E}{4} y_1^2 \right) \exp\{y_1\} \sin x, \quad (15')$$

$$\psi_1 = \left(k_1 y_1 + \frac{E}{4} y_1^2 \right) \exp\{y_1\} \cos x, \quad (17')$$

where

$$p_0 = p_2 - \frac{\rho_0 g L}{2\pi} y_1;$$

$$D_1 = -\frac{\beta T_0 a p_2 L}{4\pi\mu}; \quad (16')$$

$$k_1 = -\frac{b_1 E}{2} + b_2 a.$$

Let us examine the relationship between the coefficients $k(k_1)$ and E (taking account of the known relation $\mu = \rho l (2RT_0/\pi)^{1/2}$):

$$\frac{k}{E} = \frac{b_1}{2} + \frac{b_2 a}{E} = \pi \frac{l}{L} + \frac{24R\pi^2}{\beta g} \frac{l^2}{L^3}, \quad (18)$$

$$\frac{k_1}{E} = -\pi \frac{l}{L} + \frac{24R\pi^2}{\beta g} \frac{l^2}{L^3}. \quad (19)$$

$(l/L) \ll 1$; the coefficient $(24R\pi^2/\beta g)$ for air is $2 \cdot 10^6$ m. Changing the pressure for given L, or L for constant pressure, different values can be obtained for the ratio between the coefficients $k(k_1)$ and E governing the slip and free convection. For the pressures 1, 0.1, and 0.01 atm, k and E are of the same order as L, respectively equal to 10^{-3}, 10^{-2}, and 10^{-1} m, hence $k = k_1 \simeq b_2 a$. It is seen from (16) and (16') that the slip is determined both by the nonisothermal surface (a) and by the presence of free convection (E), however, for $(b_1 E/2b_2 a) \sim 1$ the relation $k \ll E$ is satisfied.

Streamlines corresponding to (17) and (17') are presented in a and b of the sketch (Fig. 1) for $k = k_1 = (1/2)E$. The flow is characterized by the period π. A "center" type singularity holds for $x_0 = \pi, 2\pi, \ldots$. For (17) $y_0 = 1 - (2k/E) + \sqrt{(1 + 4k^2/E^2)}$, and for (17') $y_{10} = -1 - (2k_1/E) \pm \sqrt{(1 + 4k_1^2/E^2)}$, i.e., there exist two centers and the stream function ψ_1 changes sign for $y_1 = -4k_1/E$. Presented in the same sketch for comparison are streamlines corresponding to no slip ($k = 0$ or $k_1 = 0$) and no convection ($E = 0$, this case has been examined in [3]).

NOTATION

u, v	are the longitudinal and transverse velocity components;
$\Delta = (\partial^2/\partial x^2) + (\partial^2/\partial y^2)$;	
μ	is the coefficient of gas viscosity;
β	is the coefficient of thermal expansion;
R	is the gas constant;
p_1, p_2	are the gas pressures at the lower and upper plates, respectively, at the temperature T_0;
f	is the coefficient of accommodation.

LITERATURE CITED

1. L. D. Landau and E. M. Lifshits, Mechanics of Continuous Media [in Russian], Gostekhizdat (1954).
2. V. G. Leitsina and N. V. Pavlyukevich, Inzh.-Fiz. Zh., 12, No. 3 (1967).
3. Ko Tamada, J. of the Physical Society of Japan, 21, No. 11 (1966).